Measuring voting power: The paradox of new members vs. the null player axiom*

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Abstract

Qualified majority voting is used when decisions are made by voters of different sizes. In such situations the voters' influence on decision making is far from obvious; power measures are used for an indication of the decision making ability. Several power measures have been proposed and characterised by simple axioms to help the choice between them. Unfortunately the power measures also feature a number of so-called paradoxes of voting power. In this paper we show that the Paradox of New Members follows from the Null Player Axiom. As a corollary of this result it follows that there does not exist a power measure that satisfies the axiom, while not exhibiting the Paradox.

Keywords and phrases: a priori voting power, paradox of new members, null player axiom.
JEL classification: C71, D72.

1 Introduction

Power measures or, more appropriately: a priory measures of voting power give an indication of a voter's ability to change decisions. Voting bodies are everywhere from faculty councils to the UN Security Council, from national parliaments to shareholders' meetings. The most discussed case is, however,

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the EU Council of Ministers that uses qualified majority voting, rather complicated procedures to determine whether a subset of EU member countries is able to pass a decision or not. A key element of these procedures is the voting weight that is determined by EU treaties for all countries. How should these weights be determined is a topic of ongoing discussions partly because the implications of a particular set of weights is not totally clear.

To illustrate the difficulties consider one of the simplest possible voting bodies (such as a shareholders' meeting) with only three decision makers respectively having 49, 49, and 2 votes (such as shares). A decision can be passed with plain majority, that is, if a coalition supporting the motion has at least 51 votes in total. It is easy to verify that despite the dramatic differences in weight (size) the three voters have equal influence on decision making as any pair of voters has majority, plus of course the grand coalition including all voters has majority, too. The problem is general: when voters have weights, these weights are difficult to translate into shares of voting power.

To complicate the arguments consider now a very similar problem, where three parties have the above number of representatives in a legislative body, such as a national parliament. When the MPs are allowed to vote freely and their preferences are largely independent of the parties, the probability that the voter that turns a losing coalition into a winning one belongs to a given party is proportional to the shares of seats the party has Kóczy (2008a).

In order to measure voting power several approaches have been proposed. The first measured voting power by directly applying the Shapley value to simple cooperative games (Shapley and Shubik, 1954). The most common alternative is the Banzhaf measure and index[1] (Banzhaf, 1965; Coleman, 1971; Penrose, 1946) which, although predates the Shapley-Shubik index, the first two discoveries have been largely forgotten and were only connected to the mainstream literature later. Since then several alternatives have been introduced: the Johnston-index (Johnston, 1978), the Deegan-Packel index (Deegan and Packel, 1978), the Public Good Index (Holler and Packel, 1983) or the partition value (Neyman and Tauman, 1979) so the question naturally arises: which one of these should be used?

The key to answering this question is to study the properties of the various indices. As we have seen in the above example the institutional design might imply one or another index. In general, however it is rather difficult to make a direct choice. On the other hand if the various indices and measures are characterised by a handful of simple, elementary properties or axioms making a choice between these axioms is often easier. In the best case a set

[1] It is common to refer to power measures normalised to 1 as *power indices*.

of axioms fully characterise an index, that is, there is a unique index that satisfies the given axioms. While in theory the idea is simple and appealing, many of the axioms used in these characterisations are rather technical and show little relevance to practical problems (Laruelle and Valenciano, 2005, pp37-38). It seems other properties, that perhaps do not help towards a full characterisation, but ones that have clearer practical implications can be equally useful to make a choice (Felsenthal and Machover, 1998, 2004). Unfortunately as researchers have produced positive results also some rather unattractive properties have been discovered. The so-called paradoxes of voting power (Brams, 1975/2003) are three rather intuitive properties that are nevertheless not satisfied by the best-known power indices, the Shapley-Shubik and the Banzhaf index. Felsenthal and Machover (1995) argue that "paradox" is perhaps a word too strong to describe these properties and argue that these are merely 'apparently strange pieces of behaviour' (Felsenthal and Machover, 1998, p. 221), but then the question arises whether we should be guided by our intuition and consider the paradoxes a problem or whether we should be comforted by the theoretical underpinnings of these indices and revise our intuition.

In an earlier paper (Kóczy, 2008a) we have shown that there is a rather intuitive index, the proportional index where none of the paradoxes arise. On the other hand there is ample empirical evidence that suggests that in practical matters this proportional index is used as a rule of thumb to assign power[2]. We believe that the paradoxes and a number of other results stem from this proportional index, but then it is somewhat difficult to decide what "natural properties" should serve as the basis of evaluation of power indices and which ones are prejudices rather than true requirements.

In this paper we show that the problem is general: an index that satisfies the widely accepted axioms will necessarily exhibit some of the paradoxes. In particular we show that the paradox of new members is in conflict with the null player axiom.

The outline of this paper is then as follows. First we introduce voting games and some of the well-known properties. Then we present the Brams's paradoxes and an axiomatisation of the Shapley-Shubik and Banzhaf indices, including the null player axiom. Next we prove our main result. We end the paper with some conclusions.

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[2]See Diermeier and Merlo (2004); Gelman, Katz, and Bafumi (2004); Fréchette, Kagel, and Morelli (2005) and references therein

2 Voting games

Since Shapley and Shubik (1954) it is common to study voting situations as cooperative games. A cooperative game is given by a pair (N, v) consisting of a set of n players and a real valued function, the so-called *characteristic function*[3] v defined over the set of *coalitions*: a coalition is a subset of the player set. Thus $v : 2^N \longrightarrow \mathbb{R}$. It is common to make a number of assumptions about the characteristic function. We assume that the empty set has no value, therefore $v(\varnothing) = 0$ and that the function is superadditive or

$$v(S \cup T) \geq v(S) + v(T) \quad \text{if} \quad S \cap T = \varnothing. \tag{1}$$

For such games the total value of the players is maximised by the grand coalition N and hence the purpose of the game is to find an equilibrium allocation of this payoff among the players. While strategies are not explicit in cooperative games (they correspond to forming coalitions), we still deal with the same intelligent, payoff-maximising agents as in noncoopertive games, and in fact the cooperative concepts used here have been implemented as equilibria of certain noncooperative games (Gul, 1989; Pérez-Castrillo and Wettstein, 2001).

In the following we shall be interested in *simple games*: A game is *simple* if the value of the coalition is either 0 or 1, that is, if $v : 2^N \longrightarrow \{0, 1\}$. As we shall see these values can be seen as wins and losses, and correspondingly we can talk about winning and losing coalitions. We denote the set of winning coalitions by $\mathcal{W}$, thus

$$\mathcal{W} = \{S \,|S \subseteq N, v(S) = 1\}.$$

In a winning coalition we will be interested in critical players, that is, players whose presence is essential for the success of the coalition. Formally the player i is critical in coalition S if $S \in \mathcal{W}$, but $S \setminus \{i\} \notin \mathcal{W}$. A player that is never critical is called *null*. Among winning coalitions, *minimal winning coalitions* containing only critical players deserve special attention. The set of such coalitions is denoted by $\mathcal{M}$, where

$$\mathcal{M} = \{S \,|S \in \mathcal{W}, \forall i \in S : \; S \setminus \{i\} \notin \mathcal{W}\}.$$

We assume that the grand coalition is always winning, that is, $v(N) = 1$ and that the addition of new members to a coalition does not make it loosing,

[3] The name comes from the early literature of game theory. Von Neumann and Morgenstern in their seminal work (von Neumann and Morgenstern, 1944) assumed that when in an n player game a subset S of players forms a coalition, this coalition must play against a natural opponent, the complementer coalition $N \setminus S$. The value the coalition S can obtain in this game is its characteristic value.

formally if $v(T) = 1$ and $T \subseteq S$ then $v(S) = 1$. While this assumption is a standard one, in some cases the addition of a new member to a winning coalition may also result an *infeasible* (winning) coalition. In such cases the coalition, though has the power to make decisions, cannot. We will use infeasible coalitions to formulate one of the paradoxes below, but this idea is used in the literature of games over convex geometries (Bilbao and Edelman, 2000; Bilbao, Jiménez, and López, 1998) and in the models of strategic power indices, where players have the ability to block coalitions (Kóczy, 2008b). The set of feasible coalitions is denoted by $\mathcal{F}$. Unless otherwise stated we assume that $\mathcal{F} = 2^N$.

A *weighted voting game* $\mathrm{G} = (N, (w_i)_{i \in N}, q)$ consists of a collection N of n *voters* having $w_1, w_2, \ldots, w_n > 0$ votes such that $w = \sum_{i=1}^{n} w_i$, and a quota q, $w \geq q > w/2$, or the number of votes *required* to pass a bill. For more on weighted voting games see Straffin (1994). It is clear that there is a unique mapping from weighted voting games to voting games, and therefore the first is a subset of the latter. Given $(N, (w_i)_{i \in N}, q)$ we define the corresponding voting game (N, v) by

$$v(S) = \begin{cases} 1 & \text{if } \sum_{i \in S} w_i \geq q \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

2.1 Power indices

Now that the games have been defined we can move on to defining the ways to establish the power of the individual players. In this respect there are two approaches depending on the goal of the study. On the one hand we can look at the players' share of power. In this case we calculate power *indices*, that is, normalised power measures. This is the approach we take here. When the focus is on engineering the voting situation, determining the voting weights and quotas or, more generally, the set of winning coalitions, the likelihood that any coalition is winning is important too. For it is good to know if one can change the decision in a certain percentage of cases, but such cases rarely occur the power is certainly more limited. Put it differently, power is often money – it is sufficient to think of lobbying to see this. If so, it is one thing the get a good slice of the cake and another to have a large cake, that is, much lobbying.

Since our focus is not on the institutional design, we present power indices.

A *power index* is a function k that assigns to each weighted voting game a non-negative vector in $\mathbb{R}^N_+$.

The *Shapley-Shubik index* (Shapley and Shubik, 1954) is an application of the Shapley value (Shapley, 1953) to measure voting power, motivated by a story where parties throw their support at a motion in some order until a winning coalition is reached. The last, *pivotal* party gets all the credit; the Shapley-Shubik index is then the proportion of orderings where it is pivotal

$$\Phi_i = \frac{\#\text{ times } i \text{ is pivotal}}{n!}.$$

There is also an explicit formula to express the Shapley-Shubik index:

$$\Phi_i = \sum_{S \ni i} \frac{(s-1)!(n-s)!}{n!} \left(v(S) - v(S \setminus \{i\})\right) \tag{3}$$

The *Banzhaf measure* (Penrose, 1946; Banzhaf, 1965) is the probability that a party is *critical* for a coalition, that is, the probability that it can turn winning coalitions into losing ones.

$$\psi_i = \frac{\#\text{ times } i \text{ is critical}}{2^{n-1}}.$$

Or explicitly:

$$\psi_i = \frac{1}{2^{n-1}} \sum_{S \ni i} \left(v(S) - v(S \setminus \{i\})\right) \tag{4}$$

The *Banzhaf index* β Coleman (1971) is the normalised Banzhaf measure, where the total power is scaled to 1 –already in the spirit of the Shapley-Shubik index.

The two indices can give substantially different implications, despite the fact that the main difference is in the probabilities they attach to the formation of particular coalitions or to the fact that a given player is critical for some coalition. While in the Banzhaf index all such instances of criticality happen with equal probability, for the Shapley-Shubik index the probability depends on the size of the coalition (when a player is critical in medium sized coalitions, this is taken with a smaller weight into account).

There are a few variants of the (normalised) Banzhaf index. In the *Johnston index* γ (Johnston, 1978) the credit a critical player gets is inversely proportional to the number of critical players in the coalition. In effect, coalitions of different sizes have the same contribution to the distribution of power or the probability that a given coalition is the one making the decision is the same for all coalitions. The *Deegan-Packel index* ρ (Deegan and Packel, 1978) is a further modification that only considers minimal winning coalitions, motivated by the idea that only minimal winning coalitions should form so that the benefits from winning should be least divided (Riker, 1962). Finally the Holler-Packel or *Public Good Index* h (Holler and Packel, 1983) modifies the Deegan-Packel index: here the benefit of forming a winning coalition is given to each and every player in the coalition. With the normalisation in simple games the index is nothing but a normalised Banzhaf index, where only minimal coalitions are taken into account. Finally the *partition index* (Neyman and Tauman, 1979) is motivated by decision making with multiple alternatives that then results in a partition of the voters that consists of possibly more than two coalitions. The probability that a coalition forms is then the probability that a partition containing this coalition forms. The partition index clearly favours smaller winning coalitions.

The *proportional index* α is the trivial power index given by $\alpha_i = \frac{w_i}{w}$. This measure is popularly known in political science as Gamson's Law: 'Any participant will expect others to demand from a coalition a share of the pay-off proportional to the amount of resources which they contribute' Gamson (1961).

2.2 Axioms

In the following we present the full characterisations of Dubey (1975) and Dubey and Shapley (1979) for the Shapley-Shubik and the Banzhaf index respectively.

Before we move to the different axioms we need to introduce some additional terminology. The permutation π of the players is a bijective mapping of the player set. The permutation of a game πv is given by $(\pi v)(S) = v(\pi(S))$.

Definition 1 (Anonymity Axiom)**.** For all simple games v any permutation π of N, and any $i \in N$,

$$k_i(\pi v) = k_{\pi(i)}(v). \tag{5}$$

Definition 2 (Null Player Axiom)**.** For any simple game v and any $i \in N$, if i is a null player in game v then

$$k_i(\pi v) = 0. \tag{6}$$

For two simple games v and w over the player set N let

$$(u \vee w)(S) = \max\{v(S), w(S)\} \quad \text{and} \quad (u \wedge w)(S) = \min\{v(S), w(S)\}.$$

Of these two conditions the latter is perhaps the more interesting one as it gives a formula for a combination game, for instance a game where a coalition must be winning in both chambers of the parliament or when there are multiple criteria to determine the winning coalitions as in the case of the EU Council of Ministers, for instance.

Definition 3 (Transfer Axiom)**.** For any simple games v, w such that $v \vee w$ is a simple superadditive game, too the transfer axiom states that

$$k_i(v) + k_j(w) = k(v \wedge w) + k(v \vee w) \tag{7}$$

Finally there are two axioms that express a notion of efficiency for both the Banzhaf and the Shapley-Shubik case.

Definition 4 (Shapley Total Power Axiom)**.** For any simple game v

$$\sum_{i \in N} \Phi_i(v) = 1 \tag{8}$$

Definition 5 (Banzhaf Total Power Axiom)**.** For any simple game v

$$\sum_{i \in N} \psi_i(v) = \frac{1}{2^{n-1}} \sum_{i \in N} \sum_{S \ni i} (v(S) - v(S \setminus \{i\})) \tag{9}$$

With these definitions the Shapley-Shubik index and Banzhaf measure can be characterised as follows:

Theorem 6 (Dubey (1975))**.** *If a power index k satisfies Anonimity, Null Player, Transfer and Shapley Total Power Axioms, then $k = \Phi$.*

That is the Shapley-Shubik index is the unique power index satisfying the above axioms.

Theorem 7 (Dubey and Shapley (1979))**.** *If a power measure k satisfies Anonimity, Null Player, Transfer and Banzhaf Total Power Axioms, then $k = \psi$.*

The Banzhaf measure is the unique power measure satisfying the above axioms.

While there are many other axiomatisations of these (and other indices), the null player axiom is one of the central properties. In fact, the proportional index is mostly criticised for not satisfying this property (for players with nontrivial weights). These motivate our interest in the Null Player Axiom.

2.3 Paradoxes

Brams (1975/2003) lists three natural properties that any power index should satisfy (the list is extended by Felsenthal and Machover (1998)), but the best-know indices satisfy none of these. These disappointing negative results are known as *paradoxes*. In the following we list them in their positive form as "properties."

Given a game (N, v) by the merger of players i and j we mean a modified game (N_{ij}, v_{ij}) with one less players $N_{ij} = N \backslash i, j \cup \{ij\}$ and winning coalitions

$$\mathcal{W}_{ij} = \left\{ S \in 2^{N_{ij}} \mid S \in \mathcal{W}, \text{ or } ij \in S \text{ and } S \setminus \{ij\} \cup \{i, j\} \in \mathcal{W} \right\}$$

When the game is defined as a weighted voting game the combined player ij has a weight $w_{ij} = w_i + w_j$.

Definition 8 (Property of (large) size)**.** Let (N, v) be a voting game and k a power index. Define (N_{ij}, v_{ij}) by the merger of players i and j. The game satisfies the *property of large size* if $k_i(v) + k_j(v) \leq k_{ij}(v')$.

Definition 9 (Property of new members)**.** Now define (N^+, v^+) as an extension of (N, v) by parties $n+1, \ldots, m$ such that $\mathcal{W} \setminus \mathcal{W}^+ = \emptyset$. The *property or new members* is satisfied if $k_i(N, v) \geq k_i(N^+, v^+)$, that is, the introduction of new members should not increase a party's power.

Starting from a voting game (N, v) consider the game (N, v^{ij}), which only differs in the fact that players i and j refuse to cooperate, thus

$$\mathcal{W}^{ij} = \{S \in \mathcal{W} \,|\, \{i, j\} \not\subset S\}$$

Definition 10 (Property of quarrelling members Riker (1962))**.** If two parties refuse to vote together, this should not increase their total power. Formally $k_h(N, v) \geq k_h(N, v^{ij})$ for $h \in i, j$.

Note that the game defined here is not a proper voting game, since necessarily $N \notin \mathcal{W}^{ij}$, so the grand coalition is not a feasible winning coalition.

2.4 Axioms and paradoxes

The properties above turn into paradoxes when we find that none of the well-known power indices satisfy all these. In fact the Shapley-Shubik and Banzhaf indices fail all three (Brams, 1975/2003). This result is well-known, and we leave it to the reader to find examples of games where the paradoxes appear. Unfortunately the paradoxes do not only appear in made-up examples, but van Deemen and Rusinowska (2003) found numerous real life instances in Dutch politics.

In fact, we show that the Null Player Axiom implies the Paradox of New Members and therefore any index based on the aforementioned axiom will be unreliable in circumstances where the player set is likely to expand. Unfortunately, yet again, such examples are common, in fact, the most common application of power measures is the EU Council of Ministers that is expected to expand further in the coming years, moreover the recent surge of interest in power indices is largely due to the "problems" caused by the extensions.

Theorem 11. *The Null Player Axiom implies the Paradox of New Members.*

Proof. Consider a power index k that satisfies the Null Player Axiom. For such an index all players that do not contribute to any of the winning coalitions receive a value of 0. Now consider an extension of the game and we show that for all games there is an extension such that a null player becomes non-null.

On the other hand the Paradox of New Members implies that there exist games that fail the Property of New Members. We therefore restrict our attention to weighted voting games of the form $(N, (w_i)_{i \in N}, q)$. Consider an arbitrary game with a null player that we denote by i. Instead of minimal winning coalitions consider maximal losing coalitions, in the following sense

$$L(i) \in \arg\max_{\substack{S \ni i \\ S \notin \mathcal{W}}} \sum_{j \in S} w_j, \tag{10}$$

That is, losing coalitions that lose with the smallest margin. Then consider the extension $(N \cup \{k\}, (w_i)_{i \in N \cup \{k\}}, q')$ where w_k and q' are determined as

$$w_k = (q' - q) + \left(q - \sum_{j \in L(i)} w_j \right) \tag{11}$$

Which can be reorganised as follows:

$$q' = w_k + \sum_{j \in L(i)} w_j. \tag{12}$$

For w_k sufficiently large (or, rather, not too small) the game is a proper simple game. On the other hand observe that the coalition $L(i) \cup \{k\}$ is a minimal winning coalition in the extended game, moreover, per definition, $i \in L(i) \cup \{k\}$. In a minimal winning coalition all players are critical, so i is critical in $L(i) \cup \{k\}$ and a player that is critical in any coalition is not null. Therefore i is not null in the extended game $(N \cup \{k\}, (w_i)_{i \in N \cup \{k\}}, q')$ and therefore its power increases as a result of a new member. Therefore the index exhibits the Paradox of New Members. □

In the more general case considering winning coalitions, but no voting weights the argument is not so easy to present formally and would rely on

Table 1: The 1958 Council of Ministers and two possible expansions.

Country	Original (%)			Expansion I (%)			Expansion II (%)		
	w_i	Φ_i	β_i	w_i	Φ'_i	β'_i	w_i	Φ''_i	β''_i
Germany	4	23.3	23.8	4	22.4	21.6	4	21.9	22.0
France	4	23.3	23.8	4	22.4	21.6	4	21.9	22.0
Italy	4	23.3	23.8	4	22.4	21.6	4	21.9	22.0
Netherlands	2	15.0	14.3	2	10.7	11.8	2	13.6	12.2
Belgium	2	15.0	14.3	2	10.7	11.8	2	13.6	12.2
Luxembourg	1	0	0	1	5.7	5.9	1	3.6	4.9
new member				1	5.7	5.9	1	3.6	4.9
Total	17	100.0	100.0	18	100.0	100.0	18	100.0	100.0
Quota	12			12			13		

some additional assumptions, but it is clear that if a new member is very weak, the coalition $\{i, k\}$ is losing, while if it is very powerful k can be a dictator. Assuming a "smooth" change in the size of k (and the corresponding update of $\mathcal{W}$) at one point the coalition $\{i, k\}$ becomes winning, while k, in itself is not yet, and then the player i is critical.

To illustrate that such extensions are not completely artificial, consider the EU Council of Ministers. While in the early days of –what is now called– the EU most decisions were made unanimously, already in the 6-member Community the rules of qualified majority voting to make decisions were laid down. The weights were specified as follows: 4 votes for each of the large members (Germany, France, Italy), 2 for the medium-sized members (Belgium, Netherlands) and 1 for the smallest country, Luxembourg. The Shapley-Shubik and Banzhaf indices are presented in Table 1. Observe that Luxembourg is a null player.

Now consider an extension by a small new member, who receives the same number of votes as Luxembourg.[4] We look at two scenarios depending on the quota in the new union. Observe that in either of these scenarios Luxembourg is not any more a null player, its power has increased. While this is a hypothetical extension, since Denmark, Ireland and the UK have joined the EU none of the players are null.

[4]This was a realistic scenario in the late 60s: Soon after the formation of the EC, Denmark, Ireland, Norway and the UK have applied. The application of the UK was vetoed by France, Norway withdrew due to a referendum, while the remaining two due to the veto on the UK. Had any of the smaller countries joined, the result would have been close to one of the above alternatives.

3 Conclusion

In democracies decisions are commonly made by (qualified majority) voting, be those decisions in a national parliament, at a shareholder meeting or university senate. When the voters are of different sizes voting weights can be used to express the differences and the voting rules can then be expressed as qualified majority voting. As soon as the voting situation is asymmetric it is not so straightforward to see the actual influence of a single voter in the voting process not to mention complex voting situations such as in the UN Security Council, where the permanent members have veto rights (Owen, 1995, Chapter XII), or the International Monetary Fund where some countries vote directly some via voting coalitions that themselves share power in varying ways (Reynaud, Lange, Gatarek, and Thimann, 2007). The topic has entered even the popular press when the extension of the EU made an update to voting in the Council of Ministers necessary. What is then the power of the individual member states? The Council uses currently a very complicated voting mechanism with three criteria, so it is hardly surprising that one needs powerful tools to evaluate this and similar voting situations.

The theory of a priory measures of voting power has developed much since Shapley and Shubik, and it does have a number of answers to such and similar questions. Unfortunately more than one answer has been given and a selection is far from obvious. The commonly used indices have been axiomatised by a number of elementary properties hoping that based on the attractiveness of simple properties a choice can be made. On the other hand a number of shortcomings of current theories have surfaced and been summarised as “paradoxes”. The word paradox may be too strong, the behaviour these properties describe is certainly odd. In this paper we have shown that the Paradox of New Members is a direct consequence of the Null Player Axiom therefore of one insists on the latter, the first is not at all paradoxical.

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